Round and Round: Modeling Vehicle Flow in Traffic Circles

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Abstract

We develop a general, simplified model to optimize the safety of traffic circles with varying entrance points, speed limits, and radii. We focus mainly on the safety within a traffic circle and treat maximizing flow as a secondary objective. In order to establish the safest conditions, yield signs are used only when a car is able to, on average, traverse the entire length of the traffic circle alone. The other signaling systems, stop signs and traffic lights, are timed so that every car entering the circle should, on average, not intercept another car traversing the circle. The choice between stop signs and traffic lights depends on which system will yield the most efficient flow rate through the traffic circle. If lights are used, our system determines how the lights will alternate safely as well as efficiently.

Using realistic parameters, our model verifies intuitions regarding the geometry of and flow through traffic circles. In general, as flow increases a greater degree of traffic control is necessary. Additionally, as the number of entrances or radius increases, the likelihood that a light system will be used increases substantially.

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1 Introduction

What is the best way to manage the intersection of a number of roads? One option is using traffic circles. The use of modern traffic circles has led to reduced delays at intersections. Additionally, many studies have found that one of the benefits of traffic circles is the improvement in overall safety performance [4].

Safety and efficiency should be the main concerns of any public project. Safety features are consistently improving in modern society, so road features must as well. We have been charged to model the best way to control traffic into a circle. Our main objective is to create the safest traffic circle with reasonable wait times. With this in mind, our model determines what type of signals should be used to get a certain number cars through an intersection in a given time period. If traffic lights are needed, our model establishes a method for alternating traffic lights safely and efficiently.

2 Assumptions and variables

We assume the following question is being asked of us by a traffic engineer:

"In a given time period, I have a certain number of cars wanting to get into a traffic circle from from a bunch of entrances. How do we safely control the traffic, and how long does it take to get all the cars through the circle?"

2.1 Simplifying assumptions

- Each entrance is assumed to be equidistant apart. Each entrance is assumed to have only one lane feeding into the circle. Entrances are treated as points instead of lanes having widths.

- All traffic signals are placed outside the traffic circle. This is so cars entering the circle yield to the traffic already in the circle. We chose to place all the signals and signs outside the circle for safety reasons. It has been shown that if drivers in the traffic circle have to yield to vehicles entering the circle, traffic became more congested; thus, our model should allow traffic to continue to flow even even at relatively high traffic volumes [3].

- Cars going through different lights enter the circle at the exact same times. There is no timing delay. This makes flow into the circle predictable.

- Since we do not know the conditions outside the circle like the speed limit or if traffic is backed up outside the exits, we assume the circle is isolated beyond the waiting line of cars.

2.2 Variable choices

We now describe all variables used in our model along with their notation.

The number of entrances feeding into the traffic circle is ϵ. The total number of cars that would like to pass through the circle in a given time period β is N. Denote

$$N = n_1 + n_2 + n_3 + \ldots + n_\epsilon$$

The number of lanes inside the traffic circles is λ. If ϵ is even, $\lambda = \frac{\epsilon}{2}$. If ϵ is odd, $\lambda = \frac{\epsilon}{2} - 1$. All lanes go the same direction, counterclockwise. This number was set for our traffic lighting scheme as well as safety. If traffic lights are the optimal signal choice, then our model has at most $\frac{\epsilon}{2}$ simultaneous green lights. Theoretically, but not practically, this means that every light has its own lane to feed into. The obvious problem with this justification is that cars have to cross other (congested) lanes to get to the middle of the circle or to exit. However, our model times lights and limits flow as if *only* the outermost lane of the traffic circle is being utilized, so we stand by our assumption.

The radius of the traffic circle is ρ. We chose not to set ρ formulaically since a traffic engineer might have more than optimizing flow in mind. If space is available, a planner might want to make the radius larger to include an aesthetically pleasing center island.

The speed limit inside the traffic circle is υ.

The average distance from the front of one car to the front of the following car within the traffic circle is γ. We assume an average car length of 5 m, so $\gamma > 5$ must hold. We compute γ from traffic data. At entrance i, the average time between cars coming into the circle is $\frac{\beta}{n_i}$. We know the speed limit inside the circle is υ, and we have assumed that as soon as a car enters the circle its speed does not deviate from υ. Thus, the average distance between two cars coming in the circle from entrance i is

$$\gamma_i = \frac{\upsilon \cdot \beta}{n_i}$$

After computing γ_i for all entrances, we take the weighted average of the

γ_is based on the share of total traffic contributed by the entrance. Hence

$$\gamma = \sum_{i=1}^{\epsilon} \frac{n_i}{N} \cdot \gamma_i = \sum_{i=1}^{\epsilon} \frac{n_i}{N} \cdot \frac{\upsilon \cdot \beta}{n_i} = \frac{\epsilon \upsilon \beta}{N} \tag{1}$$

Using this γ, we determine if yield signs would be an efficient way to move traffic through the circle. If traffic lights or stop signs need to be used, we need a way to estimate γ. To do this, we do not treat the traffic as flowing and set $\gamma = 10$. See Section 5 for further discussion.

If yield signs are not suggested, we need to establish a lighting system and how it operates. We split the entrances into two (three if the number of entrances are odd) groups. To establish how long each group has green lights, we use time shares based on the ratio of cars waiting at a group of lights to the total number of cars waiting. Set ν_1 to be the time share for one group of lights to cycle. ν_2 is the time share for the other group of lights to cycle. If ϵ is odd, we obviously cannot split the lights into two equal groups. In this case, we split $\epsilon - 1$ cars into two groups. We allow the final entrance to have its own lighting cycle. This is the light with the most cars waiting. We call this share ν_3.

The distance between lights that are green at the same time is α.

$$\alpha = 2 \cdot \frac{2\pi\rho}{\epsilon}$$

Since the entrances are equidistance apart, the separation between entrances with green lights is $2 \cdot \frac{2\pi\rho}{\epsilon}$. The first car out of each green light is just about to reach the next green light when the lights turn red. In essence, our flow condition allows the outermost lane of the traffic circle to fill (one car every 10 m). While red, all the cars can make it out of the circle. It might seem strange that we have more than one inside lane in our circles since we are assuming we only fill the outermost lane. We know that in reality, cars do not follow each other at a constant speed and distance. Although we assume this for calculations, inner lanes allow this to be violated in practice without destroying our model. If cars need to disperse, there is room in the inner lanes.

The length of a green light should allow the outermost lane of the circle to fill up. The length of a red light must be enough for the last car through the green light to make it to whatever exit necessary without being obstructed. Since we do not know how far this last car has to go, in general we assume the worst that the car has to make a complete loop. With these conditions, we get the following variables:

ψ is the amount of time a light is green in one light cycle:

$$\psi = \frac{\alpha}{\upsilon}$$

ω is the amount of time a light is red in one light cycle:

$$\omega = \psi \cdot \frac{\epsilon}{2}$$

since to make a complete loop, a car takes

$$\frac{2\pi\rho}{\upsilon} = \epsilon \cdot \frac{2\pi\rho}{\epsilon}\frac{1}{\upsilon} = \omega$$

Thus, the length of one light cycle δ is equal to $\omega + \psi$.

To calculate our model's flow, we have φ, the number of cars through green lights in one light cycle. If ϵ is even, we have

$$\varphi_e = \frac{\epsilon}{2} \cdot \frac{\alpha}{\gamma} = \frac{2\pi\rho}{10}$$

If ϵ is odd, we have

$$\varphi_o = \frac{\epsilon - 1}{2} \cdot \frac{\alpha}{\gamma}$$

The derivation of φ is a key part of our model. It places restrictions on the allowable flow, making our model less efficient in that regard. However, in our model, we know that every car entering the circle can safely get out. We are willing to sacrifice delays for this guarantee.

Finally, τ is the approximate time needed to clear the intersection using traffic lights.

2.3 Variable ranges

After researching how traffic circles are engineered, we believe reasonable ranges for the following variables are:

- ϵ should be an integer between 3 and 12. We do not want to fathom circles with more than 12 entrances.

- υ should lie between 5.4 m/s and 11.1 m/s [3].

- Center islands should have a diameter of 13 to 60 meters [2]. US freeway lane widths are set at 3.6 m [1], so the radius of a traffic circle should lie in the (approximate) range of $7 + 3.6\left(\frac{\epsilon}{2}\right)$ and $30 + 3.6\left(\frac{\epsilon}{2}\right)$.

In our model, the traffic engineer can set both the radius of the traffic circle and the speed limit. Therefore, the engineer must remember to make the radius of the circle sufficiently large. A driver in the innermost lane should be able to maintain the speed limit without sliding out of the lane. Also, the radius needs to be large enough to accommodate the required number of lanes.

Variable	Description
ϵ	entrances feeding into the traffic circle
N	total cars that want to pass through the circle
β	given time period (s)
λ	lanes inside the circle
ρ	radius of the circle (m)
υ	speed limit inside the circle (m/s)
γ	distance between cars in the circle (m)
α	distance between green lights (m)
ψ	amount of time a light is green in one cycle (s)
ω	amount of time a light is red in one cycle (s)
δ	length of one light cycle (s)
φ_e	number of cars through in one cycle (ϵ even)
φ_o	number of cars through in one cycle (ϵ odd)
τ	time lights need to clear the intersection (s)
ν_i	time share for a group i of lights to cycle

Table 1: Summary of Variables

3 Graphical demonstration of the model

An important aspect of our model is light timing. To be clear, we are not using light cycle in the traditional sense. To make this idea more concrete, we walk through a simple example.

Suppose there is an intersection with four entrances and four traffic lights lights. We have included Figure 1 as an aid. Our model groups these lights into two groups, say Group A and Group B. In this example, two traffic lights are in Group A and two traffic lights are Group B. In general, we have $\frac{\epsilon}{2}$ lights in a group for ϵ even and $\frac{\epsilon-1}{2}$ for ϵ odd. Now, one group at a time

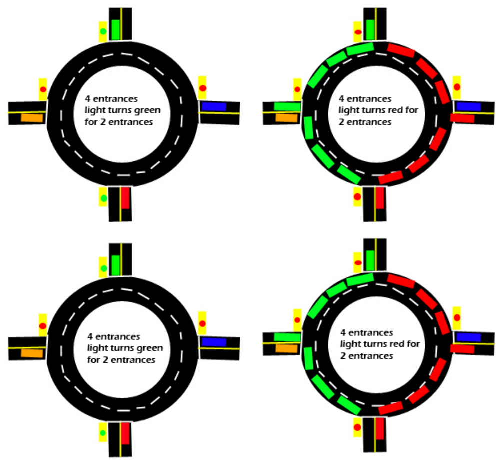

Figure 1: Traffic circle at a four lane intersection

has its lights cycling back and forth between red and green while the other group's lights are red. The concept of cycling is important. One light cycle is the amount of time for the two working lights to go from green to red back to green. Note that all four lights are red at the same time in every light cycle in our setup. We do this so that every car that enters the circle while the light is green has time to make it out before more cars enter. While all four lights are red, the cars in the circle are getting out. Once enough cycles have passed to allow all the cars in Group A through the circle, the lights at the entrances for Group B start cycling until all those cars are through.

Our algorithm does not specify how cycles should be distributed between A and B, it simply calculates how many cycles are needed for each. Nonetheless, with these numbers it would be easy to set up a cycling pattern between groups.

4 The model

We have two decisions to make. The first is what type of traffic signal to use: yield signs, stop signs or traffic lights. To simplify the analysis, we assume that the same signal is placed at every entrance. Thus, our model has a planner use only traffic lights or only stop signs or only yield signs but no mixtures. The second decision to make only concerns the use of traffic lights. If lights are used, how should they be timed? We will answer these questions in order.

4.1 When to use yield signs

To suggest a signaling system, our program needs the number of entrances to the circle being designed, the radius of the circle, the speed limit inside the circle and the vector of cars flowing through each entrance over a given time period. We propose the following condition:

If $\gamma \geq 2\pi\rho$, then the planner should use yield signs at every entrance.

This condition states that if, on average, the distance between cars entering the circle is greater than the circumference of the circle, it is safe to use yield signs. No stopping is needed. This is rational since every car could immediately enter the circle and expect to be alone while moving to their exit. Using (1), we can get a condition that only uses input values. Thus, we can restate the condition as: If $\beta \geq \frac{2\pi\rho N}{\epsilon v}$, then the planner should use yield signs at every entrance. Thus, the planning of our roundabout is done.

4.2 Timing the lights

The planning becomes much more complicated if $\beta < \frac{2\pi\rho N}{\epsilon v}$. If this is the case, we think we must control traffic for safety reasons since on average, two cars would like to be in the circle at the same time. Although two cars could safely be in the circle at once, in general we do not not know when a car is entering, when a car is already in the circle and where that car is in the circle. Thus, this is the best condition we could come up with while trying ensure safety.

Now, we are going to start stopping cars using traffic lights or stop signs. By assumption, we set $\gamma = 10$. See Section 2.2 for this justification. To determine how long it will take to get N cars through the circle using our lighting system, we need to know if ϵ is even or odd. This is because we have one more group of lights that needs to cycle in an odd system. We will show how we derive the light timing for a circle with an even number of entrances. We will simply state the result for the odd case.

To start, we need to get N cars through a circle with ϵ, an even integer, entrances. There is a light at each entrance. We have two groups of lights. One cycles between green and red while the other group stays red. In each group, $\frac{\epsilon}{2}$ lights are green at once. To make the notion of a group more concrete, one group contains $n_1 + n_3 + \ldots + n_{e-1}$ cars while the other group contains $n_2 + n_4 + \ldots + n_e$ cars.

The maximum number of cars we are allowing into the circle in one cycle is constant. This constant is

$$\varphi_e = \frac{\epsilon}{2} \cdot \frac{\alpha}{\gamma} = \frac{2\pi\rho}{10}$$

If φ_e cars can make it through the circle in one light cycle, then if we multiply φ_e by the number of times the first light group cycles added to the number of times the second light group cycles, we should get our N. Now, if we take the number of times the first light group cycles added to the number of times the second light group cycles and multiply it by the cycle length δ, we should get τ, the time traffic lights need to clear the intersection, what we need for comparison.

At this point, we have

$$N = \frac{t}{\delta} \cdot \frac{2\pi\rho}{10} = \frac{t}{\omega + \psi} \cdot \frac{2\pi\rho}{10}$$

After substituting and rearranging, we get our main result:

$$\tau = 10 \cdot N \cdot \frac{\epsilon + 2}{\epsilon \upsilon} \tag{2}$$

Thus it takes τ seconds to clear an intersection with an even number of lanes given ϵ, ρ, υ, β and N. If ϵ is odd, then we get:

$$\tau = 10 \cdot N \cdot \frac{\epsilon + 2}{(\epsilon - 1)\upsilon} \tag{3}$$

Once we have τ, we can calculate the distribution of lighting times (aggregate over τ, not distributed over τ) by using our time shares ν_1 and ν_2. The first group of lights gets $\nu_1 \cdot \tau$ seconds to cycle and the second group gets $\nu_2 \cdot \tau$. We know the cycle length δ, so we can calculate how many times each group cycles:

$$\frac{\nu_i \cdot \tau}{\delta}$$

We also know the amount of time each cycle is green, ψ. With these facts, we can determine how many times each light needs to turn green and for how long each light needs to stay green to get N cars through the circle. However, the distribution is not specified. We could let all cars in the first group clear out before letting anyone from the second group through the circle, but this would not be fair. Thus, the engineer needs to partition $\nu_i \cdot \tau$ into a certain number of intervals and alternate letting groups through the circle. This will not get all the cars through faster, but it will let cars through in a "fairer" pattern.

4.3 Choosing between traffic lights and stop signs

We now know how much time it takes traffic lights to move N cars through a traffic circle. We now establish guidelines for choosing between this lighting system and the use of stop signs. We want to compare the the rate at which cars enter the traffic circle using traffic lights, $\frac{\tau}{N}$ seconds per car, to some rate that stop signs could allow. This rate is calculated as follows. It takes $\frac{2\pi\rho}{\upsilon} \cdot \frac{1}{\epsilon}$ seconds for an entering car to make it to the next entrance. We have λ free lanes in each circle. Thus, stop signs can allow a rate of

$$\frac{2\pi\rho}{\epsilon\upsilon} \cdot \frac{1}{\lambda}$$

seconds per person safely into the circle. This way, we can put one car into each free lane at every entrance. These cars are then able to make it to the next entrance before others enter the circle, so we consider the circle "safe." Hence, we only use stop signs at the entrances if the following condition holds:

$$\frac{2\pi\rho}{\epsilon\upsilon} \cdot \frac{1}{\lambda} < \frac{\tau}{N} \tag{4}$$

Our model thus provides answers to both questions being asked. We have conditions determining what type of signal is to be used. If lights are used, we show how they should be timed.

5 Limitations

Given the focus on safety for our model, there are some limitations.

- Our model moves cars through at a steady rate that may be inefficient. We have timed lights and stop signs so that all the cars in a circle can safely make it to their desired exit before more cars are let in. We want to avoid the possibility of one car entering and hitting another. Our model is extremely restrictive in this way since this is obviously not always necessary. Nonetheless, to make the model general, we assume the worst conditions possible for entering cars.

  In defense of our model, at very small roundabouts "it is reasonable to assume...a vehicle may not enter the circulatory roadway unless the quadrant on both sides of the approach is empty." [2] This is similar to how we allowed cars to flow into the circle.

- If entrances have a different number of cars, there will be green lights at an entrance with no cars coming through. Our model is not dynamically timed, but it can be preset for certain time intervals. Thus, a planner can still use our model to change light timings during rush hour.

- If traffic lights or stop signs need to be used, we need a way to estimate γ. To do this, we do not treat the traffic as flowing. Instead, since we are stopping cars, we treat them as a long line waiting at a light. Thus, we lower the average distance between cars when they finally can enter the circle. Since cars are entering the circle from a line, we set $\gamma = 10$. This implies there is one car length between cars at rest. This obviously limits our model. If we need to stop cars, we make the assumption that the flow of traffic becomes one long line of equally spaced cars. This greatly influences the timing of cars entering the circle, especially since it deviates from the input data. However, our analysis necessitates an assumption about how the spacing of cars would change once they are stopped.

- Another limitation of our model is how cars move. Due to the fact that we were not modeling for efficiency, we reduced the complications that can be found in other models for flow into a traffic circle such as merging time. Additionally, we assume the speed limit in the circle is reached instantaneously, and we also assume the cars keep going this speed limit. We do not account for acceleration or deceleration. Thus, cars can turn into and out of the circle both unobstructed and at the speed limit without losing control. Although we have factored out acceleration, we argue that the speed limit range is low, so acceleration would not take long.

6 Results

When used, the maximum number of cars per hour our lighting system can handle is

$$\frac{\epsilon \upsilon}{10(\epsilon + 2)} \cdot 3600 \cdot 0.85 \tag{5}$$

The factor of 0.85 was included because successful traffic circle operation takes place when the volume-to-capacity ratio does not exceed 0.85 [2]. This condition places bounds on how effective our circle is at moving through a cars. Once out of range, our circle becomes less and less effective.

We ran model simulations using realistic data in comparison to modern roundabouts. These simulations serve an expository purpose, for our inputs (entrances, radius, flow, speed limit) are within range of general traffic circle

Intersection	ϵ	ρ	υ	β	N	Max cars/hr	Signage:
Freeway Change	4	25	11	3600	950	N/A-low flow	Yields
Large Urban	8	45	9	3600	1000	2203	Stop signs
Suburban Street	3	13	7	3600	100	N/A-low flow	Yields
Rush Hour Street	4	20	11	3600	2200	2244	Lights

Table 2: Model Simulation Results

characteristics [4]. They show what our model suggests given differing sets of parameters. Table 2 depicts various scenarios and possible inputs for each situation. For more precise data, especially for calculating the timing of lights, the reader can turn to our MATLAB functions included in the Appendix.

7 Discussion

From our simulation results, we can see that our model makes intuitive sense by verifying when different controls should be used. For example, urban circles are expected to experience heavy traffic. Urban conditions typically have more entrances, which implies larger circles and a slightly reduced speed limit. We would expect that more control would be necessary since there is heavy traffic and a faster paced environment. Also, suburban street circles typically have small radii, lower speed limits and low traffic flow. Therefore, not as much control is needed as there is not as much traffic. Finally, traffic lights were suggested when we simulated a rush hour intersection. This situation occurs frequently. The estimated time to clear the cars was almost an hour since the vehicles/hour were close to maximum capacity.

In general, we verified intuitions regarding the geometry and flow through traffic circles. As flow increases, a greater degree of traffic control is necessary. Additionally, the geometry of the circle affects the flow. Figure 2 demonstrates how the number of entrances affects the traffic flow (seconds/car to enter the circle) holding the speed limit constant ($\upsilon = 8$). For the equations used, see (4). The blue line is the traffic flow when lights are used. The black lines are the flow of traffic using stop signs. Since stop sign flow varies with radius of the circle, different radii are plotted. As you move along the x-axis in the positive direction, the radius of the circle is

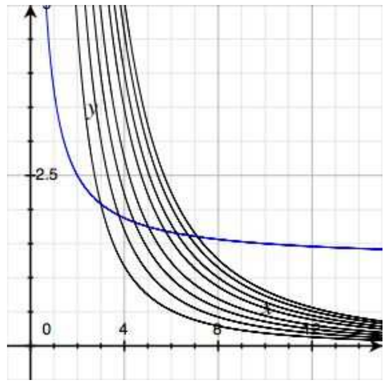

Figure 2: Entrances versus traffic flow (secs/car)

increasing.

Figure 2 shows the importance of the geometric shape of the circle. As the circle gets smaller, traffic lights become less helpful. This makes sense. If a small circle is being used, hopefully not a lot of cars need to get through the circle. Thus, stop signs provide enough control. However, if the radius is large, there must be a large number of entrances for stop signs to be useful. If there are fewer entrances, cars must stay in the circle longer to get to an exit. For safety reasons, we would have to suggest controlling the traffic more. Traffic lights accomplish this.

8 Conclusion

Our task was to develop a model that determines the type of traffic flow control to be used for various traffic circles. Focusing on safety and then efficiency, we designed a program that determines the traffic control method that should be used in different situations. If lights are necessary, the program outputs precisely how to alternate between green and red. We successfully tested our model with several scenarios and realized that the results were reasonable although, admittedly, many simplifying assumptions were made. Our models greatest strength is the adaptability of the model for various situations and as stated before, the minimization of accidents. In conclusion, as long as traffic flow is reasonable, we believe our model will accurately determine the type of traffic control to use.

9 Appendix: Algorithm for determining signal type and light timing

9.1 MATLAB Code for Even Number of Entrances

```
function [t]=even_opt(e,r,v,f,N)
   % receive a umber of entrances to the traffic circle e
   % radius of traffic circle r
   % entering speed limit v
   % vector N of number of cars hitting each entry point
   % to the circle in a time period f
   % Function will output the suggested signal and time
   % t a safe traffic light system
   % would need to get all the cars N through the circle
   % start with some basic sums
   oddsum=0;
   for i=1:2:e-1
   oddsum=oddsum+N(i);
   end;
   evensum=0;
   for i=2:2:e
   evensum=evensum+N(i);
   end;
   totalcars=oddsum+evensum;
   oddshare=oddsum/totalcars;
   evenshare=evensum/totalcars;
   % compute weighted avg distance between entering cars
   % for each entrance
   C=[e:1];
   avgc=0;
   for i=1:e
   C(i)=(v*f)/N(i);
   avgc=avgc+C(i)*N(i)/totalcars;
   end;
   % check to see if yielding is more efficient
   yieldcheck=2*pi*r*totalcars/(e*v);
   if f>=yieldcheck
   u=3 % use a yield sign
   % if not, need to calculate light times and
   % and compare with stop signs
```

```
else
% calculate time needed to get all cars through using lights
avgc=10;
t=totalcars*avgc*(e+2)/(e*v);
% not how long lights are green, how long lights cycling
oddlightson=t*oddshare; % share of time for light groups
evenlightson=t*evenshare;
cyclelength=((e+2)/2)*(2*2*pi*r/(e*v))
% number of cycles for a group
cyclesodd=oddlightson/cyclelength
% amount of time a light is green for a group
greenodd=(2*pi*r/(e*v))/cyclelength*oddlightson;
cycleseven=evenlightson/cyclelength
greeneven=(2*pi*r/(e*v))/cyclelength*evenlightson;
oddflow=cyclesodd*2*pi*r/avgc;
evenflow=cycleseven*2*pi*r/avgc;
totalflow=oddflow+evenflow
lighttime=t
%determine the max veh/h capacity of our model
max=(2*pi*r/10)/((4*pi*r/(e*v))*((e+2)/2))*3600*.85
compare=t/totalflow;
stopcomp=(2/e)*(2*pi*r)/(v*e);
% if sec/car is less for stop signs than lights, use those!
if stopcomp<compare
u=2 % use a stop sign
else
u=1 % use traffic lights
end
end
```

9.2 MATLAB Code for Odd Number of Entrances

```
function [t]=odd_opt(e,r,v,f,N)
   % when timing lights, one entrance goes through alone
   % Make this the heaviest traffic lane.
   alone=max(N);
   newN=[e-1:1];
   index=1;
   % find index of max
   for i=1:e
```

```
comp=N(i);
if comp==alone
index=i;
end
end
% drop max entrance from array
for i=1:e
if i<index
newN(i)=N(i);
end
if i>index
newN(i-1)=N(i);
end
end
% some basic sums
% firstsum and secondsum are pairs nonadjacent lights
firstsum=0;
for i=1:2:e-1
firstsum=firstsum+newN(i);
end;
secondsum=0;
for i=2:2:e
secondsum=secondsum+newN(i);
end;
totalcars=firstsum+secondsum+alone;
aloneshare=alone/totalcars;
firstshare=firstsum/totalcars;
secondshare=secondsum/totalcars;
% compute weighted avg distance between entering cars within
% circle for each entrance
C=[e:1];
avgc=0;
for i=1:e
C(i)=(v*f)/N(i);
avgc=avgc+C(i)*N(i)/totalcars;
end;
% check to see if yielding is more efficient
yieldcheck=2*pi*r*totalcars/(e*v);
if f>=yieldcheck
u=3
```

```
else
% if not, need to calculate light times and
% compare with stop signs
% calculate time needed to get all cars through
avgc=10;
t=(totalcars*avgc*(e+2)/((e-1)*v));
alonelight=t*aloneshare;
firstlight=t*firstshare;
secondlight=t*secondshare;
cyclelength=((e+2)/2)*(2*2*pi*r/(e*v))
cyclesalone=alonelight/cyclelength
alonegreen=alonelight/((e+2)/2);
cyclesfirst=firstlight/cyclelength
firstgreen=firstlight/((e+2)/2);
cyclessecond=secondlight/cyclelength
secondgreen=secondlight/((e+2)/2);
aloneflow=cyclesalone*2*pi*r*(e-1)/(avgc*e);
firstflow=cyclesfirst*2*pi*r*(e-1)/(avgc*e);
secondflow=cyclessecond*2*pi*r*(e-1)/(avgc*e);
totalflow=firstflow+secondflow+aloneflow
lighttime=t
%determine the max veh/h capacity of our model
max=(2*pi*r/10)/((4*pi*r/(e*v))*((e+2)/2))*3600*.85
compare=t/totalflow;
stopcomp=(2/(e-1))*(2*pi*r)/(v*e);
% if sec/car is less for stop signs than lights, use those!
if stopcomp<compare
u=2
else
u=1
end
end
```

References

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9 February 2009

RE: Traffic Circle Conundrum

To whom it may concern,

As per your request, we have developed a model using MATLAB for controlling car flow in traffic circles. Our model's main objective is safety. We achieved this by ensuring that no car will be in front of a turning car in our model. Our model determines what method of traffic flow control to use based on the radius, speed limit, number of entrances and volume of cars waiting.

The three methods of traffic flow control that we used are yields, stop signs and traffic lights. Variables that need be input into our model are radius of the circle, speed limit, number of entrances, model run time and volume of cars waiting. These can be chosen or observed empirically. Once these variables are determined, our model will output the type of traffic control. If our model suggests the use of traffic lights, the model will also output the time the lights should be green and how long they should be red. Our model will also output the expected time to get all the cars waiting through the traffic circle.

Using Table 1 as a reference, we propose the following condition for yield sign use: If

$$\frac{\epsilon\upsilon\beta}{N} \geq 2\pi\rho,$$

then use yield signs at every entrance. Generally speaking, yield signs would be used if on average the distance between cars entering the circle is greater than the radius of the circle, it is safe to use yield signs. If yield signs are not to be used, we use the following for considering stop sign use over traffic light use: If

$$\frac{2\pi\rho}{\epsilon\upsilon} \cdot \frac{1}{\lambda} < \frac{\tau}{N},$$

then use stop signs at every entrance; otherwise, use traffic lights. Thus, if the time it takes per car to enter the traffic circle using traffic lights is less than the time it takes per car to enter the traffic circle using stop signs, then our model states that traffic lights should be used. When traffic lights are used, the model determines how long the lights should be green based on the time it takes one car to reach another entering lane.

Variable	Description
ϵ	entrances feeding into the traffic circle
N	total cars that want to pass through the circle
λ	lanes inside the circle
ρ	radius of the circle (m)
υ	speed limit inside the circle (m/s)
β	given time period (s)
τ	time lights need to clear the intersection (s)

Table 1: Summary of Variables

Using these criteria, we verified intuitions regarding the geometry and flow through traffic circles. As flow increases, a greater degree of traffic control is necessary. Additionally, the geometry of the circle affects the flow. As the circle gets smaller, traffic lights become less helpful in controlling flow. This makes sense. If a small circle is being used, hopefully not a lot of cars need to get through the circle. Thus, stop signs provide enough control. However, if the radius is large, there must be a large number of entrances for stop signs to be useful. If there are fewer entrances, cars must stay in the circle longer to get to an exit. For safety reasons, we would have to suggest controlling the traffic more. Traffic lights accomplish this.

Attached to this letter is our report regarding the results of the model.